New Trends in Coded Waveform Design for Radar Applications

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Abstract

In this lecture, we provide an overview of some recent algorithms for radar waveform design. We first illustrate the concept of matched illumination for optimized target detection and identification. This theory derives the optimized transmission signal and the corresponding receiver response which maximizes the Signal to Noise Ratio (SNR).

The second part of the lecture is focused on waveform optimization in the presence of colored disturbance with known covariance matrix. Focusing on the class of coded pulse trains, we present a code selection algorithm which maximizes the detection performance but, at the same time, is capable of controlling both the region of achievable values for the Doppler estimation accuracy and the degree of similarity with a pre-fixed radar code.

Finally, the last part of the talk is devoted to the challenging issue of Multiple-Input Multiple-Output (MIMO) radar waveform design. In this context, we discuss the benefits of waveform diversity enabling the MIMO radar superiority over its phased array counterpart.

Keywords. Coded Waveform Design, Matched Illumination, MIMO Radar.

I. INTRODUCTION

The huge advances in high speed signal processing hardware, digital array radar technology, and the requirement of better and better radar performances has promoted, during the last two decades, the development of very sophisticated algorithms for radar waveform design [1].

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The synthesis of narrowband waveforms with a specified ambiguity function is considered in [2]. In [3], the theory of wavelets is exploited for the design of radar signals capable of operating in a dense target environment. The use of information theory to devise waveforms for the detection of extended radar targets, exhibiting resonance phenomena, is studied in [4]. Therein, two design problems with constraints on signal energy and duration are solved.

The concept of matched-illumination for optimized target detection and identification has also received a considerable attention [6], [8], [9, and references therein] due to the advent of adaptive radar transmitters which permit the use of advanced pulse shaping techniques. This theory derives the optimized transmission waveform and the corresponding receiver response which maximizes the SNR. The case of single polarimetric channel is considered in [8], whereas the most general situation of full polarization radar transmission is handled in [9]. The concept is also further investigated in [10], with reference to Gaussian point-target and stationary Gaussian distributed clutter. The results show that the optimum procedure places the signal energy in the noise band having minimum power.

Waveform optimization in the presence of colored disturbance with known covariance matrix has been addressed in [11]. Three techniques based on the maximization of the SNR are introduced and analyzed. Two of them also exploit the degrees of freedom provided by a rank deficient disturbance covariance matrix. In [12], a signal design algorithm relying on the maximization of the SNR under a similarity constraint with a given waveform is proposed and assessed. The solution potentially emphasizes the target contribution and de-emphasizes the disturbance. Moreover, it also preserves some characteristics of the desired waveform. In [13], a signal subspace framework, which allows the derivation of the optimal radar waveform (in the sense of maximizing the SNR at the output of the detector) for a given scenario, is presented under the Gaussian assumption for the statistics of both the target and the clutter.

A quite different signal design approach relies on the modulation of a pulse train parameters (amplitude, phase, and frequency) in order to synthesize waveforms with some specified properties. This technique is known as the radar coding and a substantial bulk of work is nowadays available in open literature about this topic. Here we mention Barker, Frank, and Costas codes which lead to waveforms whose ambiguity functions share good resolution properties both in range and Doppler. This list is not exhaustive and a comprehensive treatment can be found in [14], [15], [16]. It is, however, worth pointing out that the ambiguity function is not the only

relevant objective function for code design in operating situations where the disturbance is not white. This might be the case of optimum radar detection in the presence of colored disturbance, where the standard matched filter is no longer optimum and the most powerful radar receiver requires whitening operations.

In [17], [18] the problem of code optimization in the presence of colored Gaussian disturbance is tackled. At the design stage, the authors focus on the class of coded pulse trains and propose a code selection algorithm which is optimum according to the following criterion: maximization of the detection performance under a control both on the region of achievable values for the Doppler estimation accuracy and on the degree of similarity with a specified radar code. Actually, this last constraint is equivalent to force a similarity between the ambiguity functions of the devised waveform and of the pulse train encoded with the pre-fixed sequence.

Additional degrees of freedom in waveform design, which permit a further optimization of the system performance, can be obtained exploiting a MIMO radar consisting of multiple sensors at both the transmitter and the receiver end [19]. In this last case, a Space-Time Coding (STC) strategy can be implemented, namely it is possible to code the transmitted signal both in time (i.e. from pulse to pulse) and in space (from a transmitting antenna to another).

The lecture is organized as follows. In Section II, we deal with the concept of matched illumination for enhanced target detection and identification. In Section III, which is the core of the lecture, we discuss the problem of code design in colored Gaussian disturbance and present the code design algorithm derived in [17] as well as some extensions. In Section IV, we address waveform design for MIMO radar with both colocated and widely spaced sensors. Finally, in Section V, we draw conclusions and outline possible future research tracks.

II. MATCHED ILLUMINATION FOR TARGET DETECTION AND IDENTIFICATION

A challenging design problem encountered in radar, sonar, and communication systems in general, is that of jointly optimizing the transmitter and receiver; given, perhaps, some knowledge of the channel [8]. In the active sensor detection problem (radar, sonar, lidar), one is concerned with judiciously selecting the operating band, transmit waveform modulations, and receiver processing strategy, in order to maximize the probability of detecting the presence of a "target", while maintaining a prescribed rate of false alarm [20], [5], [6], and [7].

In the case of radar, a first generation of so-called "matched illumination-reception" systems
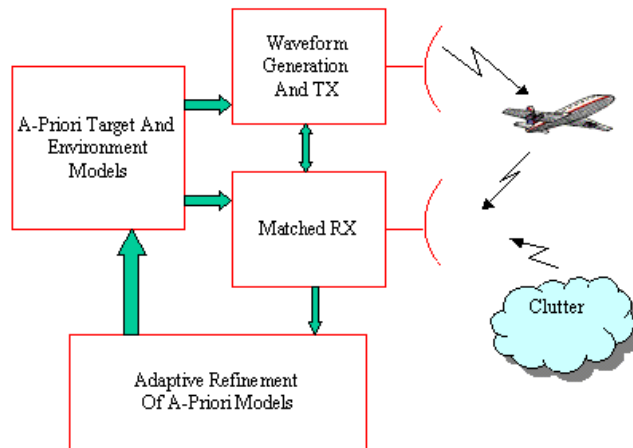
Figure 1: Matched illumination-reception scheme.

(see Figure 1) has been designed relaxing the "point target" assumption, usually met in several current surveillance radars. Specifically, the optimal waveform is obtained as an eigenfunction of a Fredholm integral operator of the first kind whose kernel is formed by the impulse response of the target. With reference to the case of Additive White Gaussian Noise (AWGN), it is well known that this solution maximizes the output SNR of an appropriately matched filter [21].

In [8], a general matched illumination theory for optimized target detection with reference to a single polarimetric channel has been developed. The authors derive the optimized transmission waveform and the corresponding receiver response which maximize the SNR in the presence of generic colored noise and signal-dependent clutter. Interestingly, optimized single-polarization waveform focuses most of its energy into one or a few narrow frequency bands [22].

The extension to the multiple channel case, which is of particular relevance for full-polarization waveforms (containing both the horizontal and vertical polarizations) is addressed in [23]. It relies on a multiple channel factorization of the noise and clutter power spectral density, in order to determine the optimized transmission waveform and the receiver response. The approach has been modified in [9], through a discrete matrix formulation, which avoids the computation of a multiple channel spectral factorization. Therein, the improvements for target detection and identification are also studied with reference to realistic simulated radar response data of tactical ground moving targets.

It is worth pointing out that the aforementioned approaches provide a bound on the maximum potential performance improvement in SNR, resulting from the use of matched illumination over standard chirp waveforms. Clearly, the practical application of these concepts in a real system requires an optimization involving multiple constraints. For instance, it is desirable that the range-sidelobes of the resulting waveform be low, in order to reduce possible range ambiguities.

III. CODE DESIGN IN THE PRESENCE OF COLORED DISTURBANCE

In this section[1], we consider the problem of radar code design in the presence of colored Gaussian noise. To this end, we consider a radar system which transmits a coherent burst of pulses

$$s(t) = a_t u(t) \exp[j(2\pi f_0 t + \phi)]\,,$$

where a_t is the transmit signal amplitude, $j = \sqrt{-1}$,

$$u(t) = \sum_{i=0}^{N-1} a(i) p(t - iT_R)\,,$$

is the signal's complex envelope, $p(t)$ is the signature of the transmitted pulse, T_R is the Pulse Repetition Time (PRT), $[a(0), a(1), \ldots, a(N-1)] \in C^N$ is the radar code (assumed without loss of generality with unit norm), f_0 is the carrier frequency, and ϕ is a random phase. Moreover, the pulse waveform $p(t)$ is of duration $T_p \leq T_R$ and has unit energy, i.e.

$$\int_0^{T_p} |p(t)|^2 dt = 1\,,$$

where $|\cdot|$ denotes the modulus of a complex number. The signal backscattered by a target with a two-way time delay τ and received by the radar is

$$r(t) = \alpha_r e^{j2\pi(f_0+f_d)(t-\tau)} u(t-\tau) + n(t)\,,$$

where α_r is the complex echo amplitude (accounting for the transmit amplitude, phase, target reflectivity, and channels propagation effects), f_d is the target Doppler frequency, and $n(t)$ is additive disturbance due to clutter and thermal noise.

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[1]The material in this section of the lecture is taken from [17], [18].

This signal is down-converted to baseband and filtered through a linear system with impulse response $h(t) = p^*(-t)$, where $(\cdot)^*$ denotes complex conjugate. Let the filter output be

$$v(t) = \alpha_r e^{-j2\pi f_0 \tau} \sum_{i=0}^{N-1} a(i) e^{j2\pi i f_d T_R} \chi_p(t - iT_R - \tau, f_d) + w(t),$$

where $\chi_p(\lambda, f)$ is defined as

$$\chi_p(\lambda, f) = \int_{-\infty}^{+\infty} p(\beta) p^*(\beta - \lambda) e^{j2\pi f\beta} d\beta,$$

and $w(t)$ is the down-converted and filtered disturbance component. The signal $v(t)$ is sampled at $t_k = \tau + kT_R$, $k = 0, \dots, N-1$, providing the observables[2]

$$v(t_k) = \alpha a(k) e^{j2\pi k f_d T_R} \chi_p(0, f_d) + w(t_k), \qquad k = 0, \dots, N-1,$$

where $\alpha = \alpha_r e^{-j2\pi f_0 \tau}$. Assuming that the pulse waveform time-bandwidth product and the expected range of target Doppler frequencies are such that the single pulse waveform is insensitive to target Doppler shift[3], namely $\chi_p(0, f_d) \sim \chi_p(0, 0) = 1$, we can rewrite the samples $v(t_k)$ as

$$v(t_k) = \alpha a(k) e^{j2\pi k f_d T_R} + w(t_k), \qquad k = 0, \dots, N-1.$$

Moreover, denoting by $\boldsymbol{c} = [a(0), a(1), \dots, a(N-1)]^T$ the N-dimensional column vector containing the code elements[4], $\boldsymbol{p} = [1, e^{j2\pi f_d T_R}, \dots, e^{j2\pi(N-1)f_d T_R}]^T$ the temporal steering vector, $\boldsymbol{v} = [v(t_0), v(t_1), \dots, v(t_{N-1})]^T$, and $\boldsymbol{w} = [w(t_0), w(t_1), \dots, w(t_{N-1})]^T$, we get the following vectorial model for the backscattered signal

$$\boldsymbol{v} = \alpha \boldsymbol{c} \odot \boldsymbol{p} + \boldsymbol{w}, \tag{1}$$

where $\odot$ denotes the Hadamard element-wise product [24].

A. Relevant Performance Measures for Code Design

In this sub-section, we introduce some key performance measures to be optimized or controlled during the selection of the radar code. As it will be shown, they permit to formulate the design of the code as a constrained optimization problem.

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[2]We neglect range straddling losses and also assume that there are no target range ambiguities.

[3]Notice that this assumption might be restrictive for the cases of very fast moving targets such as fighters and ballistic missiles.

[4]$(\cdot)^T$ is the transpose operator.

Detection Probability. This is one of the most important performance measures which radar engineers attempt to optimize. We just remind that the problem of detecting a target in the presence of observables described by the model (1) can be formulated in terms of the following binary hypotheses test

$$
\begin{cases}
H_0: & \boldsymbol{v} = \boldsymbol{w} \\
\\
H_1: & \boldsymbol{v} = \alpha \boldsymbol{c} \odot \boldsymbol{p} + \boldsymbol{w}
\end{cases}
\tag{2}
$$

Assuming that the disturbance vector $\boldsymbol{w}$ is a zero-mean complex circular Gaussian vector with known positive definite covariance matrix

$$
E[\boldsymbol{w}\boldsymbol{w}^\dagger] = \boldsymbol{M}\,,
$$

($E[\cdot]$ denotes statistical expectation and $(\cdot)^\dagger$ conjugate transpose), the Generalized Likelihood Ratio Test (GLRT) detector for (2), which coincides with the optimum test (according to the Neyman-Pearson criterion) if the phase of α is uniformly distributed in $[0, 2\pi[$ [25], is given by

$$
|\boldsymbol{v}^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p})|^2 \underset{H_0}{\overset{H_1}{\gtrless}} G\,,
\tag{3}
$$

where G is the detection threshold set according to a desired value of the false alarm Probability (P_{fa}). An analytical expression of the detection Probability (P_d), for a given value of P_{fa}, is available both for the cases of non-fluctuating and fluctuating target. In the former case (NFT),

$$
P_d = Q\left(\sqrt{2|\alpha|^2(\boldsymbol{c} \odot \boldsymbol{p})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p})}, \sqrt{-2\ln P_{fa}}\right),
\tag{4}
$$

while, for the case of Rayleigh fluctuating target (RFT) with $E[|\alpha|^2] = \sigma_a^2$,

$$
P_d = \exp\left(\frac{\ln P_{fa}}{1 + \sigma_a^2(\boldsymbol{c} \odot \boldsymbol{p})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p})}\right),
\tag{5}
$$

where $Q(\cdot, \cdot)$ denotes the Marcum Q function of order 1. These last expressions show that, given P_{fa}, P_d depends on the radar code, the disturbance covariance matrix and the temporal steering vector only through the SNR, defined as

$$
\mathrm{SNR} = \begin{cases}
|\alpha|^2(\boldsymbol{c} \odot \boldsymbol{p})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p}) & \mathrm{NFT} \\
\\
\sigma_a^2(\boldsymbol{c} \odot \boldsymbol{p})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p}) & \mathrm{RFT}
\end{cases}
\tag{6}
$$

Moreover, P_d is an increasing function of SNR and, as a consequence, the maximization of P_d can be obtained maximizing the SNR over the radar code.

Doppler Frequency Estimation Accuracy. The Doppler accuracy is bounded below by Cramer-Rao Bound (CRB) and CRB-like techniques which provide lower bounds for the variances of unbiased estimates. Constraining the CRB is tantamount to controlling the region of achievable Doppler estimation accuracies, referred to in the following as $\mathcal{A}$. We just highlight that a reliable measurement of the Doppler frequency is very important in radar signal processing because it is directly related to the target radial velocity useful to speed the track initiation, to improve the track accuracy [26], and to classify the threat target level in defense applications. In this subsection, we introduce the CRB for the case of known α, the Modified CRB (MCRB) [27] and the Miller-Chang Bound (MCB) [28] for random α, and the Hybrid CRB (HCRB) [29, pp. 931-932] for the case of a random zero-mean α.

Proposition I. The CRB for known α (Case 1), the MCRB and the MCB for random α (Case 2 and Case 3 respectively), and the HCRB for a random zero-mean α (Case 4) are given by [17]

$$\Delta_{CR}(f_d) = \frac{\Psi}{2\dfrac{\partial \boldsymbol{h}^\dagger}{\partial f_d} \boldsymbol{M}^{-1} \dfrac{\partial \boldsymbol{h}}{\partial f_d}}, \tag{7}$$

where $\boldsymbol{h} = \boldsymbol{c} \odot \boldsymbol{p}$,

$$\Psi = \begin{cases} \dfrac{1}{|\alpha|^2} & \text{Case 1} \\[2ex] \dfrac{1}{E[|\alpha|^2]} & \text{Cases 2 and 4} \\[2ex] E\left[\dfrac{1}{|\alpha|^2}\right] & \text{Case 3} \end{cases} \tag{8}$$

Notice that

$$\frac{\partial \boldsymbol{h}}{\partial f_d} = T_R \, \boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u} \,,$$

with $\boldsymbol{u} = [0, j2\pi, \ldots, j2\pi(N-1)]^T$, and (7) can be rewritten as

$$\Delta_{CR}(f_d) = \frac{\Psi}{2T_R^2 (\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u})^\dagger \boldsymbol{M}^{-1} (\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u})} \,. \tag{9}$$

As already stated, forcing an upper bound to CRB, for a specified Ψ value, results in a lower bound on the size of $\mathcal{A}$. Hence, according to this guideline, we focus on the class of radar codes complying with the condition

$$\Delta_{CR}(f_d) \leq \frac{\Psi}{2T_R^2 \delta_a}\,, \tag{10}$$

which can be equivalently written as

$$(\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u}) \geq \delta_a\,, \tag{11}$$

where the parameter δ_a rules the lower bound on the size of $\mathcal{A}$. Otherwise stated, suitably increasing δ_a, we ensure that new points fall in the region $\mathcal{A}$, namely new smaller values for the estimation variance can be theoretically reached by estimators of the target Doppler frequency.

Similarity Constraint. Designing a code which optimizes the detection performance does not provide any kind of control to the shape of the resulting coded waveform. Precisely, the unconstrained optimization of P_d can lead to signals with significant modulus variations, poor range resolution, high peak sidelobe levels, and more in general with an undesired ambiguity function behavior. These drawbacks can be partially circumvented imposing a further constraint on the sought radar code. Precisely, it is required the solution to be similar to a known code $\boldsymbol{c}_0$ ($\|\boldsymbol{c}_0\|^2 = 1$)[5], which shares constant modulus, reasonable range resolution and peak sidelobe level. This is tantamount to imposing that [12]

$$\|\boldsymbol{c} - \boldsymbol{c}_0\|^2 \leq \epsilon\,, \tag{12}$$

where the parameter $\epsilon \geq 0$ rules the size of the similarity region. In other words, (12) permits to indirectly control the ambiguity function of the considered coded pulse train: the smaller ϵ the higher the degree of similarity between the ambiguity functions of the designed radar code and of $\boldsymbol{c}_0$.

B. Code Design Algorithm.

In this sub-section, we present a technique (proposed in [17] and [18]) for the selection of the radar code which attempts to maximize the detection performance but, at the same time,

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[5] $\|\cdot\|$ denotes the Euclidean norm of a complex vector.

provides a control both on the target Doppler estimation accuracy and on the similarity with a given radar code. To this end, we first observe that

$$(\boldsymbol{c} \odot \boldsymbol{p})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p}) = \boldsymbol{c}^\dagger \left[\boldsymbol{M}^{-1} \odot (\boldsymbol{p}\boldsymbol{p}^\dagger)^* \right] \boldsymbol{c} = \boldsymbol{c}^\dagger \boldsymbol{R} \boldsymbol{c} \tag{13}$$

and

$$(\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u})^\dagger \boldsymbol{M}^{-1}(\boldsymbol{c} \odot \boldsymbol{p} \odot \boldsymbol{u}) = \boldsymbol{c}^\dagger \left[\boldsymbol{M}^{-1} \odot (\boldsymbol{p}\boldsymbol{p}^\dagger)^* \odot (\boldsymbol{u}\boldsymbol{u}^\dagger)^* \right] \boldsymbol{c} = \boldsymbol{c}^\dagger \boldsymbol{R}_1 \boldsymbol{c}, \tag{14}$$

where $\boldsymbol{R} = \boldsymbol{M}^{-1} \odot (\boldsymbol{p}\boldsymbol{p}^\dagger)^*$ and $\boldsymbol{R}_1 = \boldsymbol{M}^{-1} \odot (\boldsymbol{p}\boldsymbol{p}^\dagger)^* \odot (\boldsymbol{u}\boldsymbol{u}^\dagger)^*$ are positive semidefinite [29, p. 1352, A.77]. It follows that P_d and $\Delta_{CR}(f_d)$ are competing with respect to the radar code. In fact, their values are ruled by two different quadratic forms, (13) and (14) respectively, with two different matrices $\boldsymbol{R}$ and $\boldsymbol{R}_1$. Hence, only when they share the same eigenvector corresponding to the maximum eigenvalue, (13) and (14) are maximized by the same code. In other words, maximizing P_d results in a penalization of $\Delta_{CR}(f_d)$ and vice versa.

Exploiting (13) and (14), the code design problem can be formulated as a non-convex optimization Quadratic Problem (QP)

$$\begin{cases} \max_{\boldsymbol{c}} & \boldsymbol{c}^\dagger \boldsymbol{R} \boldsymbol{c} \\ \text{s.t.} & \boldsymbol{c}^\dagger \boldsymbol{c} = 1 \\ & \boldsymbol{c}^\dagger \boldsymbol{R}_1 \boldsymbol{c} \geq \delta_a \\ & \|\boldsymbol{c} - \boldsymbol{c}_0\|^2 \leq \epsilon \end{cases}$$

The feasibility of the problem, which not only depends on the parameters δ_a and ϵ but also on the pre-fixed code $\boldsymbol{c}_0$, is discussed in [17]. Therein, a technique to solve (QP) has also been proposed. It ensures a polynomial computational complexity and is based on the relaxation of the original problem into a Semidefinite Program (SDP). Specifically, the best code is determined through a rank-one [31] decomposition of an optimal solution of the relaxed problem.

C. Performance Analysis

The present sub-section shows the performance of the considered encoding scheme. To this end, we assume that the disturbance covariance matrix is exponentially shaped with one-lag correlation coefficient $\rho = 0.8$, i.e.

$$\boldsymbol{M}(i, j) = \rho^{|i-j|},$$

and set P_{fa} of the receiver (3) to 10^{-6}. The analysis is conducted in terms of P_d, region of achievable Doppler estimation accuracies, and ambiguity function of the coded pulse train which results exploiting the proposed algorithm, i.e.

$$|\chi(\lambda, f)| = \left| \int_{-\infty}^{\infty} u(\beta) u^*(\beta - \lambda) e^{j2\pi f\beta} d\beta \right| = \left| \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \bar{a}(l) \bar{a}^*(m) \chi_p[\lambda - (l-m)T_R, f] \right|,$$

where $[\bar{a}(0), \ldots, \bar{a}(N-1)]$ is an optimum code. As to the temporal steering vector $\boldsymbol{p}$, we set the normalized Doppler frequency[6] $f_d T_r = 0$. The convex optimization MATLAB toolbox SElf-DUal-MInimization (SeDuMi) [32] is exploited for solving the SDP relaxation. The rank-one decomposition of the SeDuMi solution is performed using the technique described in [31]. Finally, the MATLAB toolbox of [33] is used to plot the ambiguity functions of the coded pulse trains. In the following, we choose as similarity sequence a generalized Barker code [14, pp. 109-113] which is a polyphase sequence whose autocorrelation function has minimal peak-to-sidelobe ratio excluding the outermost sidelobe. Examples of such sequences were found for all $N \leq 45$ [34], [35] using numerical optimization techniques. In the simulations of this subsection, we assume $N = 7$ and set the similarity code equal to the generalized Barker sequence $\boldsymbol{c}_0 = [0.3780, 0.3780, -0.1072 - 0.3624j, -0.0202 - 0.3774j, 0.2752 + 0.2591j, 0.1855 - 0.3293j, 0.0057 + 0.3779j]^T$.

In Figure 2a, we plot P_d of the optimum code (according to the proposed criterion) versus $|\alpha|^2$ for several values of δ_a, $\delta_\epsilon = \left(1 - \frac{\epsilon}{2}\right)^2 = 0.01$, and for non-fluctuating target. In the same figure, we also represent both the P_d of the similarity code as well as the benchmark performance, namely the maximum achievable detection rate (over the radar code), given by

$$P_d = Q\left(\sqrt{2|\alpha|^2 \lambda_{max}(\boldsymbol{R})}, \sqrt{-2 \ln P_{fa}}\right), \tag{15}$$

where $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue of the argument.

The curves show that increasing δ_a we get lower and lower values of P_d for a given $|\alpha|^2$ value. This was expected since the higher δ_a the smaller the feasibility region of the optimization problem to be solved for finding the code. Nevertheless, the proposed encoding algorithm usually ensures a better detection performance than the original generalized Barker code.

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[6]We have also considered other values for the target normalized Doppler frequency. The results, not reported here, confirm the performance behavior showed in the next two subsections.
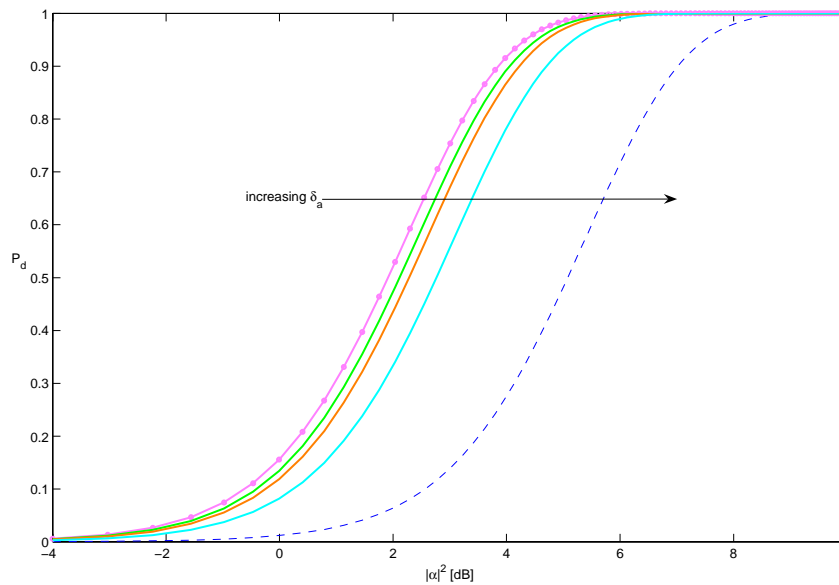
Figure 2a: P_d versus $|\alpha|^2$ for $P_{fa} = 10^{-6}$, $N = 7$, $\delta_\epsilon = 0.01$, non-fluctuating target, and several values of $\delta_a \in \{10^{-6}, 6165.5, 6792.6, 7293.9\}$. Generalized Barker code (dashed curve). Code which maximizes the SNR for a given δ_a (solid curve). Benchmark code (dotted-marked curve). Notice that the curve for $\delta_a = 10^{-6}$ perfectly overlaps with the benchmark P_d.
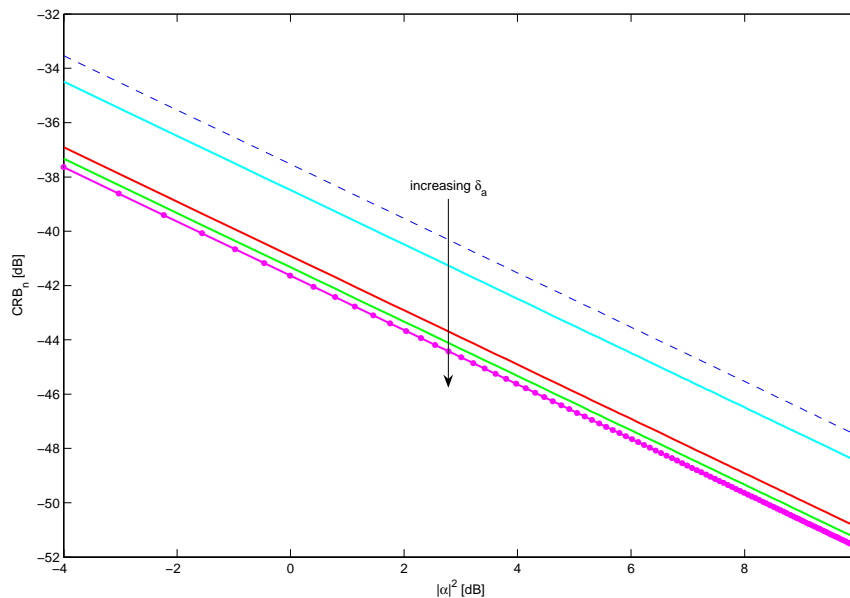

Figure 2b: CRB_n versus $|\alpha|^2$ for $N = 7$, $\delta_\epsilon = 0.01$ and several values of $\delta_a \in \{10^{-6}, 6165.5, 6792.6, 7293.9\}$. Generalized Barker code (dashed curve). Code which maximizes the SNR for a given δ_a (solid curve). Benchmark code (dotted-marked curve). Notice that the curve for $\delta_a = 7293.9$ perfectly overlaps with the benchmark CRB_n.
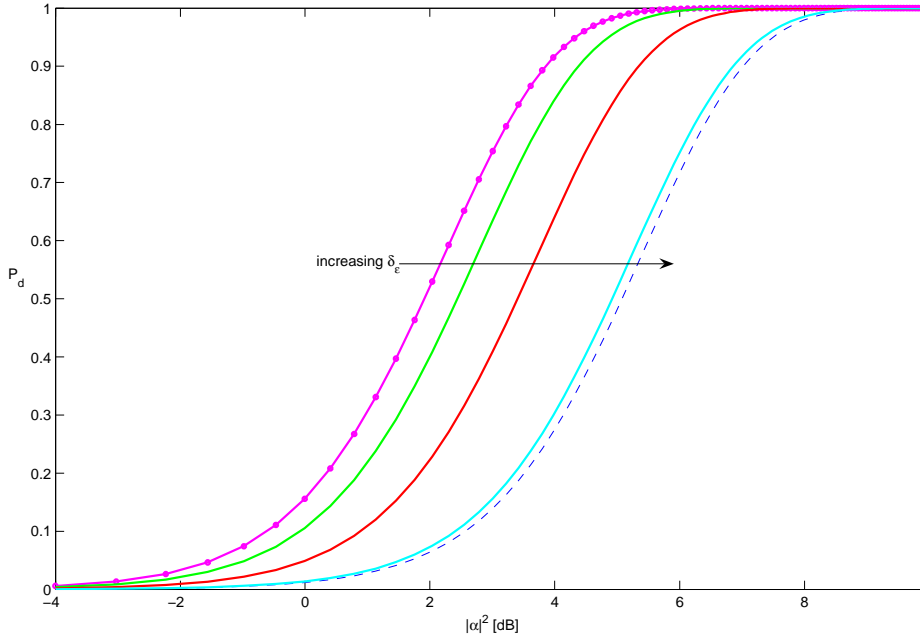
Figure 2c: P_d versus $|\alpha|^2$ for $P_{fa} = 10^{-6}$, $N = 7$, $\delta_a = 10^{-6}$, non-fluctuating target, and several values of $\delta_\epsilon \in \{0.01, 0.6239, 0.8997, 0.9994\}$. Generalized Barker code (dashed curve). Code which maximizes the SNR for a given δ_ϵ (solid curve). Benchmark code (dotted-marked curve). Notice that the curve for $\delta_\epsilon = 0.01$ perfectly overlaps with the benchmark P_d.

In Figure 2b, the normalized CRB ($\mathrm{CRB}_n = T_R^2 \mathrm{CRB}$) is plotted versus $|\alpha|^2$ for the same values of δ_a as in Figure 2a. The best value of CRB_n is plotted too, i.e.

$$\mathrm{CRB}_n = \frac{1}{2|\alpha|^2 \lambda_{max}(\boldsymbol{R}_1)}.\tag{16}$$

The curves highlight that increasing δ_a better and better CRB values can be achieved. This is in accordance with the considered criterion, because the higher δ_a the larger the size of the region $\mathcal{A}$. Summarizing, the joint analysis of Figures 2a-2b shows that a tradeoff can be realized between the detection performance and the estimation accuracy. Moreover, there exist codes capable of outperforming the generalized Barker code both in terms of P_d and size of $\mathcal{A}$.

The effects of the similarity constraint are analyzed in Figure 2c. Therein, we set $\delta_a = 10^{-6}$ and consider several values of δ_ϵ. The plots show that increasing δ_ϵ worse and worse P_d values are obtained; this behavior can be explained observing that the smaller δ_ϵ the larger the size of the similarity region. However, this detection loss is compensated for an improvement of the coded pulse train ambiguity function. This is shown in Figures 3b-3e, where such function is plotted assuming rectangular pulses, $T_R = 5T_p$ and the same values of δ_a and δ_ϵ as in Figure 1c.
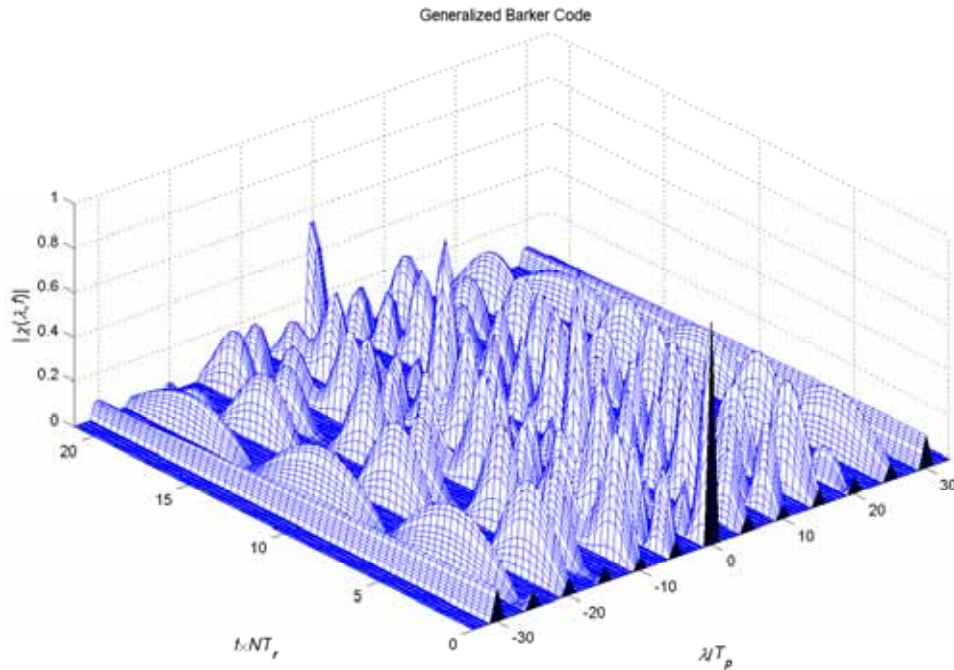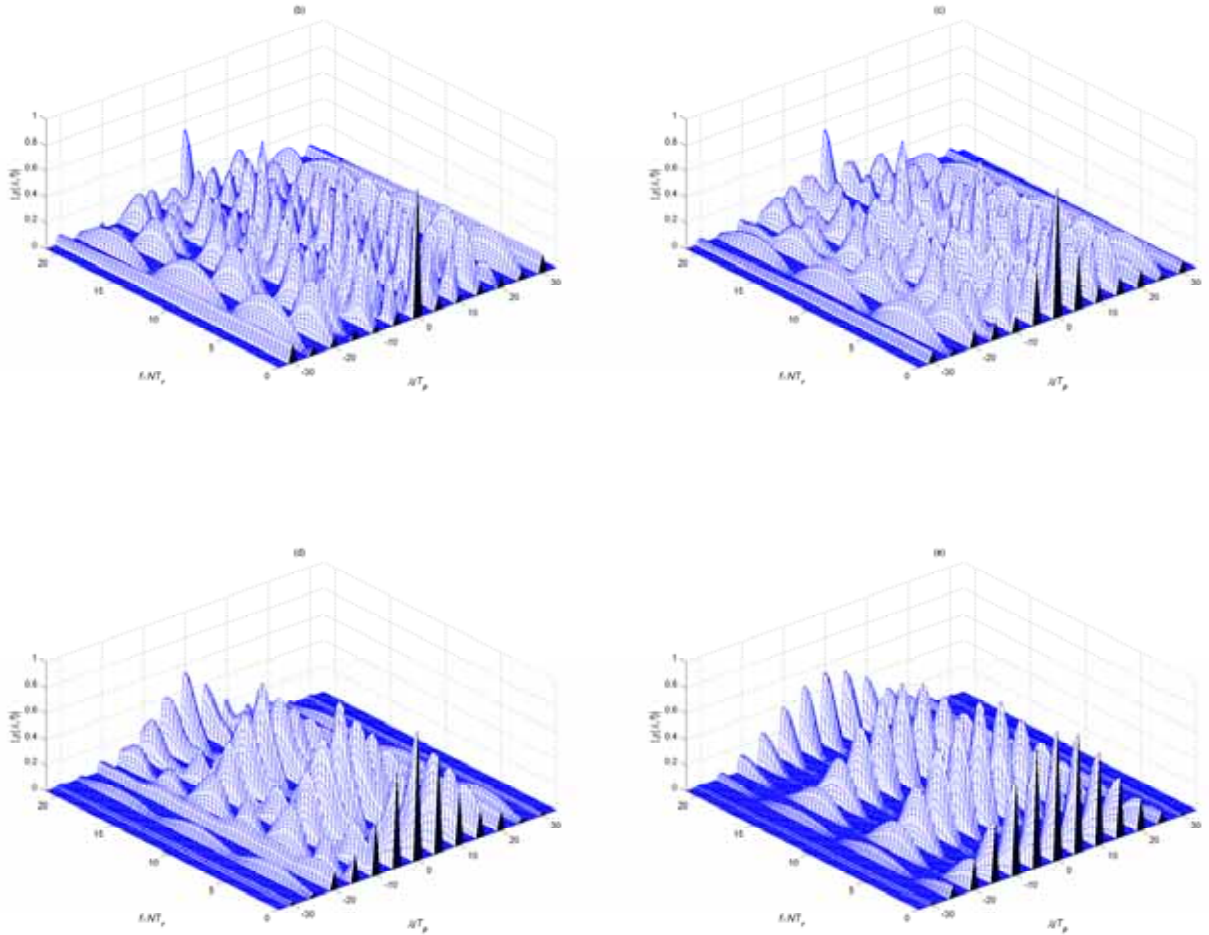
Figure 3a: Ambiguity function modulus of the generalized Barker code $\boldsymbol{c}_0 = [0.3780, 0.3780, -0.1072 - 0.3624j, -0.0202 - 0.3774j, 0.2752 + 0.2591j, 0.1855 - 0.3293j, 0.0057 + 0.3779j]^T$.

Moreover, for comparison purposes, the ambiguity function of $\boldsymbol{c}_0$ is plotted too (Figure 3a). The plots highlight that the closer δ_ϵ to 1 the higher the degree of similarity between the ambiguity functions of the devised and the pre-fixed codes. This is due to the fact that increasing δ_ϵ is tantamount to reducing the size of the similarity region. In other words, we force the devised code to be similar and similar to the pre-fixed one and, as a consequence, we get similar and similar ambiguity functions.

Figures 3b-3e: Ambiguity Function modulus of code which maximizes the SNR for $N = 7$, $\delta_a = 10^{-6}$, $\mathbf{c}_0$ generalized Barker code, and several values of δ_ϵ: (b) $\delta_\epsilon = 0.9994$, (c) $\delta_\epsilon = 0.8997$, (d) $\delta_\epsilon = 0.6239$, (e) $\delta_\epsilon = 0.01$.
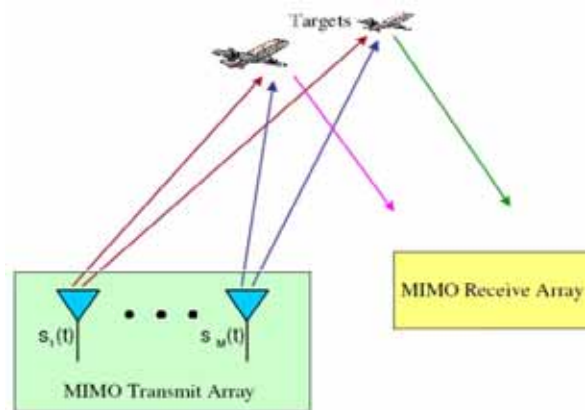
Figure 4a: MIMO with colocated antennas.

IV. MIMO RADAR WAVEFORM DESIGN

A MIMO radar is a system which exploits multiple antennas at both the transmitter and the receiver end. The sensors can be either colocated or widely spaced. The former case is usually denoted as MIMO radar with colocated antennas (Figure 4a). The latter is referred to as statistical MIMO radar (see Figure 4). MIMO radar with colocated antennas has been thoroughly studied in [19, ch. 1]. Therein, the authors focus on the merits of the waveform diversity allowed by transmit and receive antenna arrays. Their results show that the said waveform diversity enables the MIMO radar superiority in several fundamental aspects, including: significantly improved parameter identifiability [36], direct applicability of adaptive arrays for target detection and parameter estimation, much enhanced flexibility for transmit beampattern design, and waveform optimality for more accurate target parameter estimation and imaging [37]. Specifically, they prove that:

1) the maximum number of targets that can be uniquely identified by the MIMO radar is up to s times that of its phased-array counterpart, where s is the number of transmit antennas [36];

2) the echoes due to targets at different locations can be linearly independent of each other, which allows the direct application of many adaptive techniques to achieve high resolution
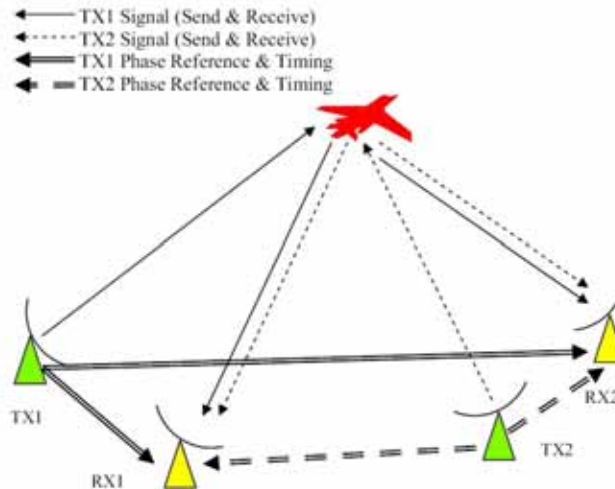
Figure 4b: Statistical MIMO radar scheme.

and excellent interference rejection capability;

3) the probing signals [38], transmitted via its antennas, can be optimized to obtain several transmit beampattern designs with superior performance;

4) the probing signals can also be optimized by considering several design criteria, including minimizing the trace, determinant, and the largest eigenvalue of the CRB matrix, to improve the radar parameter estimation and imaging performance.

On a different philosophy are based statistical MIMO radars. They aim at increasing the detector sensitivity by sufficiently spacing the sensors at both the transmitter and the receiver end, thus generating a number of independent channels, i.e. spatial diversity. It is indeed well known that target amplitude fluctuations may have a very detrimental effect in the interest region of close-to-one detection rates, while being beneficial in the region of low detection probabilities. In this context, a MIMO radar should be in principle able to provide the receiver with as many equivalent diversity branches as possible, which, in turn, requires stating the conditions under which a "rich scattering environment" can be re-created. Major contributions in this direction have been offered in [39], [40], and [41], wherein the theoretical background for importing the MIMO concept into radar was laid out, and the interplay between the physical parameters of the target to be detected, those of the transmitted signal, and the transmit/receive architectures

have been first investigated. The main results can be summarized as follows:

1) On a non-dispersive channel, and due to the abrupt fluctuations of targets RCS with the aspect angle, a MIMO radar is able to grant angle diversity, provided that time differences across the transmit sensors are preserved at the receiver end (waveform-orthogonality-preserving assumption);

2) The angle diversity yields a diversity order given by the product of the number of transmit and receive antennas, under additive white Gaussian noise disturbance. In these conditions, waveform orthogonality allows adopting a very simple path-combination rule;

3) In view of the above results, MIMO structures are superior with respect to their natural competitors, i.e. phased-array, Multiple-Input Single-Output (MISO), and Single-Input Multiple-Output (SIMO) radars, as far as detection performance is considered.

The development of a consistent theory for statistical MIMO radar has been the focus of [42], [43], [44], and [45] where, assuming a general model for the clutter correlation properties, STC has been recognized as a key ingredient to achieve full diversity, and design criteria for both the transmitter and the receiver have been given: in the new framework, the usage of orthogonal waveforms has been shown to be one possible design choice, and only a member of the general class of space-time coded radar waveforms, that can be employed for performance optimization purposes under diverse scenarios.

Before concluding this section, we highlight that MIMO radar are receiving a vibrant attention in the context of Over The Horizon (OTH) radar signal processing. In [46], the authors show how HF skywave radar performance and flexibility can benefit from transmission of multiple orthogonal waveforms in MIMO architecture. However, there are several practical limitations to account for. One issue is that many HF radar transmit arrays are over-sampled spatially to allow for operation over a significant portion of the HF band. Transmission of orthogonal waveforms, in this case, can result in large reactive power and consequent equipment damage. Another crucial issue is the ability to generate orthogonal waveform sets with sufficient cardinality at the low time-bandwidth products typical of aircraft surveillance operation [46]. Finally a demonstration of MIMO OTH radar, based on the concept of a-posteriori transmitter beamforming, is presented in [47]

V. Conclusions

In this lecture, we have presented some recent developments in the theory of radar waveform design. First of all, we have discussed the potential improvements in the output SNR which can result through the exploitation of the matched illumination concept, both with reference to the single channel and multiple channel (full-polarization) processing. Then, we have discussed the problem of radar signal design in the presence of colored Gaussian disturbance. Precisely, focusing on the class of coded pulse trains, we have presented a code selection algorithm which maximizes the detection performance but, at the same time, is capable of controlling both the region of achievable values for the Doppler estimation accuracy and the degree of similarity with a pre-fixed radar code. Actually, this last constraint is equivalent to force a similarity between the ambiguity functions of the devised waveform and of the pulse train encoded with the pre-fixed sequence. The resulting optimization problem belongs to the family of non-convex quadratic programs. In order to solve it, a relaxation of the original problem into a convex one which belongs to the SDP class is used.

In the last part of the lecture, we have introduced the challenging MIMO radar concept. The cases of both colocated and widely spaced sensors are presented. Moreover, the relevant performance benefits, achievable exploiting a MIMO structure, have been highlighted.

Possible future research tracks, might concern:

- the implementation on the field of matched illumination techniques;

- the design and the analysis of coding techniques, in colored Gaussian disturbance, capable of being adaptive with respect to the disturbance covariance matrix;

- the study of the interplay between ambiguity function and detection performance in the context of statistical MIMO radar.

We hope that this lecture stimulates the investigation toward the challenging and fertile research topic of radar waveform design.

Acknowledgements

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